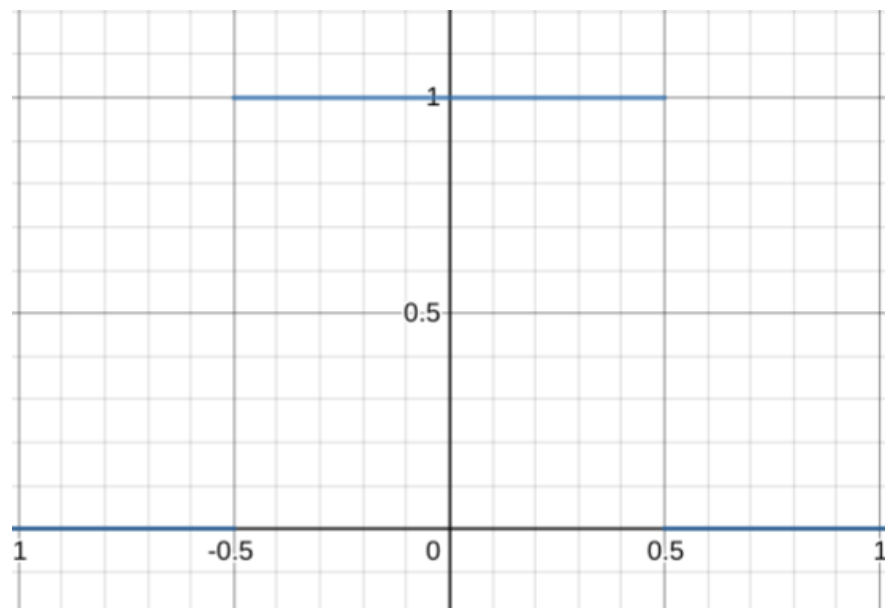


Задача 5.4(а)

Студент 303 группы ВМК МГУ
Мордвинцев Михаил

Дискретизация времени

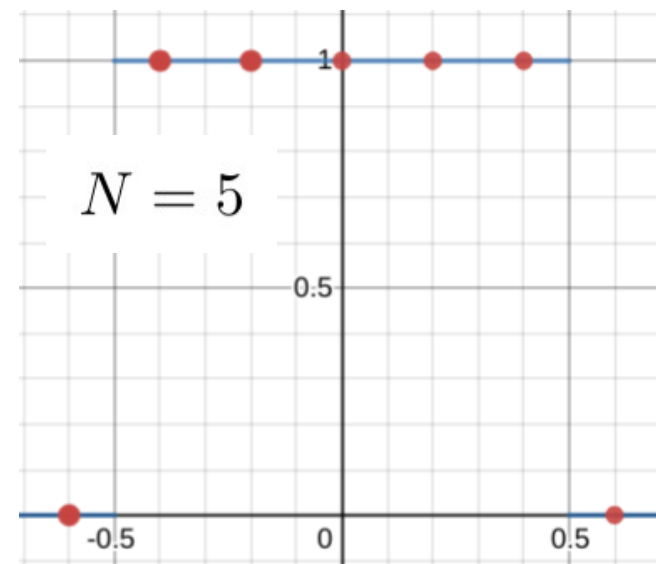
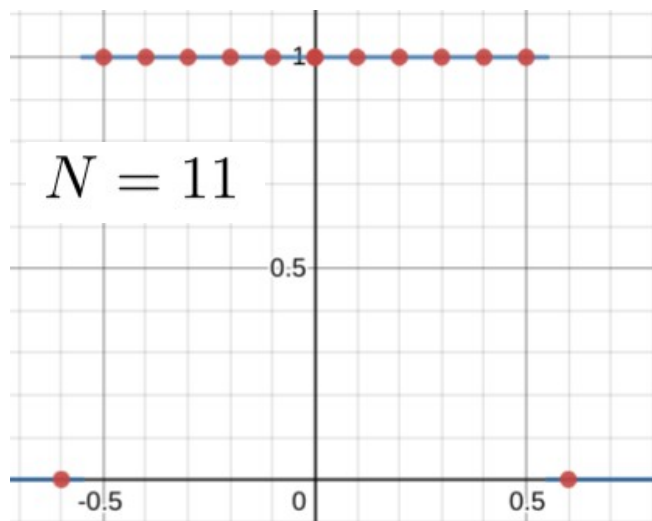
Сигнал: $\xi(t) = \begin{cases} a, & |t| < \tau/2 \\ 0, & |t| > \tau/2 \end{cases}$ Дискретизация времени с шагом $h = \frac{\tau}{N}$:



$$\begin{aligned} \xi_h(j) &= \int_{hj - \frac{h}{2}}^{hj + \frac{h}{2}} \xi(t) dt = \\ &= \begin{cases} ah, & |t| < \frac{\tau}{2} \\ 0, & |t| > \frac{\tau}{2} \end{cases} \end{aligned}$$

Дискретизация времени

$$\xi_h(j) = \begin{cases} ah, & |t| < \frac{\tau}{2} \\ 0, & |t| > \frac{\tau}{2} \end{cases} \quad h = \frac{\tau}{N}$$



Вычисление спектра

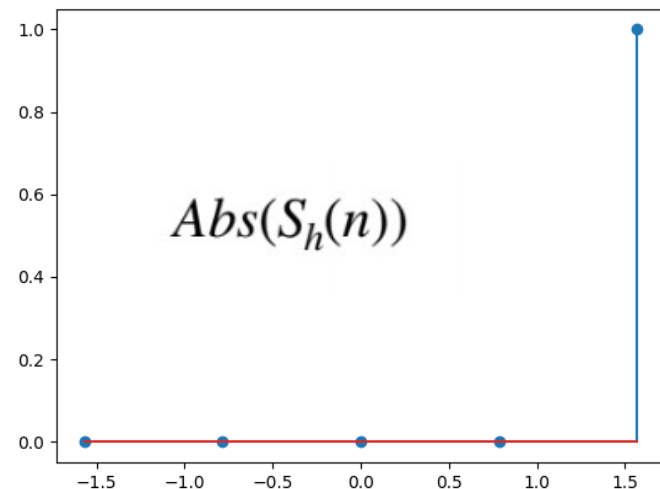
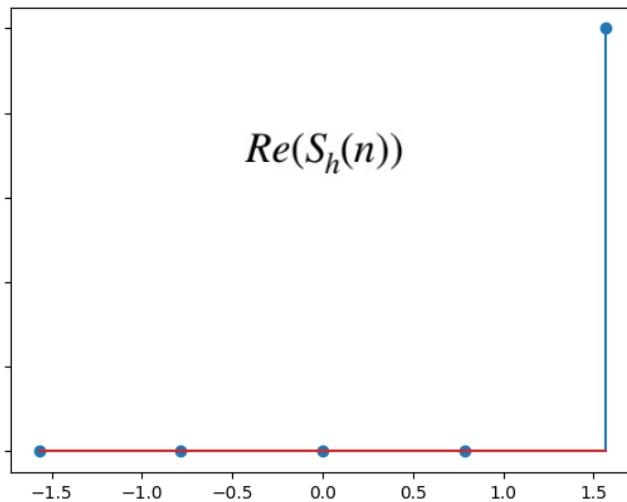
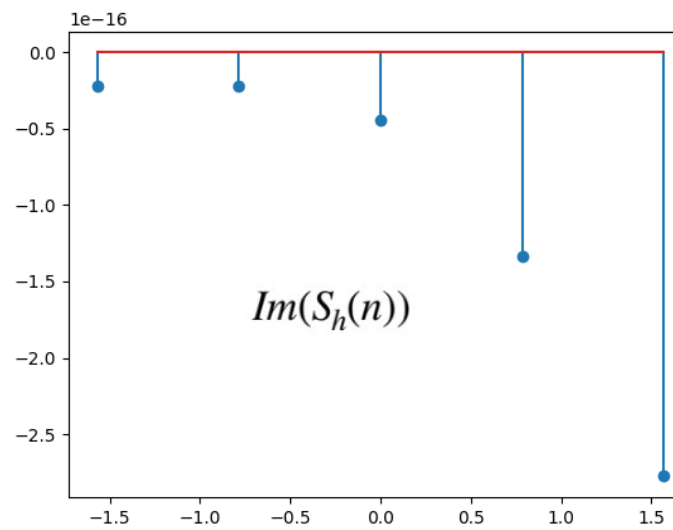
Вычисление спектра $S_h(n)$ по сигналу $\xi_h(j)$

$$S_h(n) = \frac{1}{N} \sum_{j=1}^N \xi_h(j) W_N^{nj}, \quad n = 1, 2, \dots, N.$$

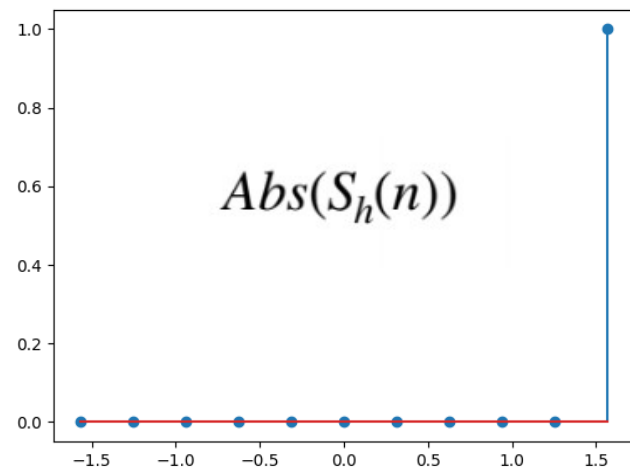
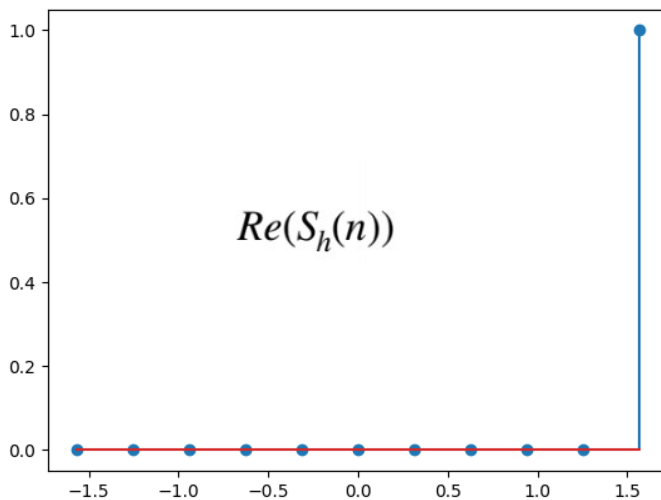
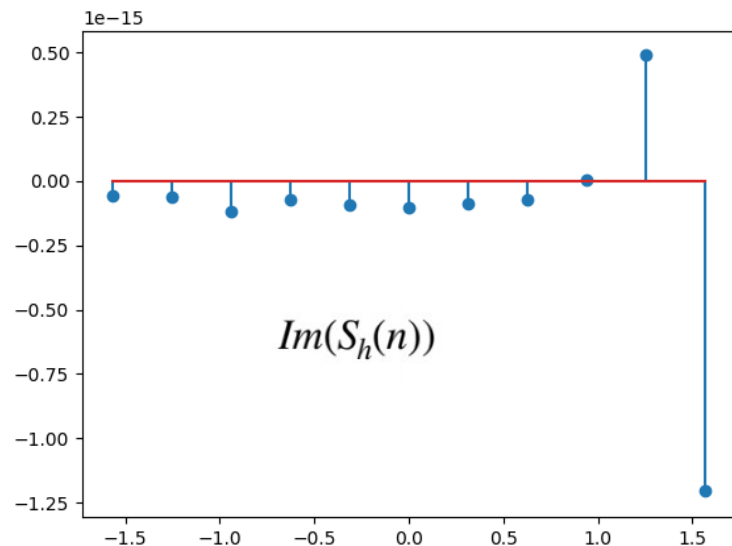
$$W_N = \exp(i \frac{2\pi}{N})$$

```
for n in range(1, N + 1):  
    for j in range(1, N + 1):  
        S[n - 1] += xi(j) * W ** (n * j)  
S[n - 1] /= N
```

Спектр: $N = 5$



Спектр: $N = 11$



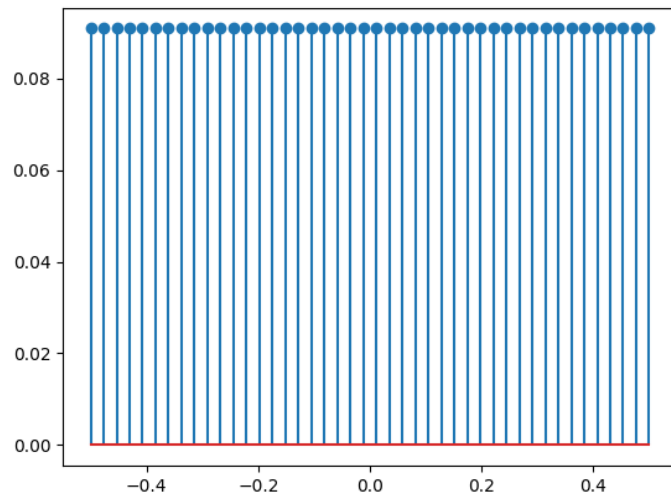
Сигнал по спектру

Вычисление сигнала $\xi_h(j)$ по спектру $S_h(n)$

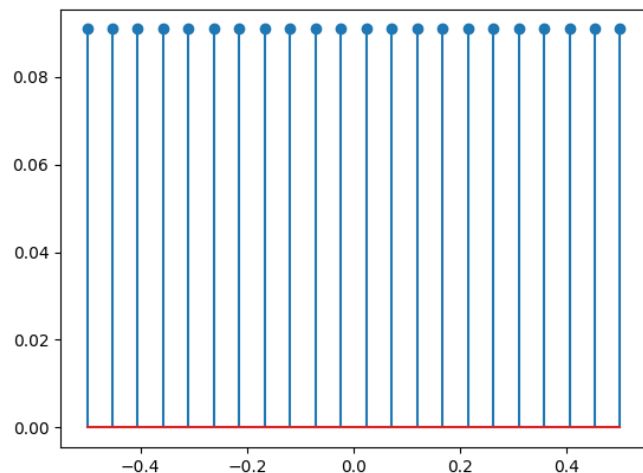
$$\xi_{h'}(j) = \sum_{n=1}^N S_{h'}(n) W_N^{-nj}, i = 1, 2, \dots, N$$

$$W_N = \exp(i \frac{2\pi}{N})$$

$h' = h/4$



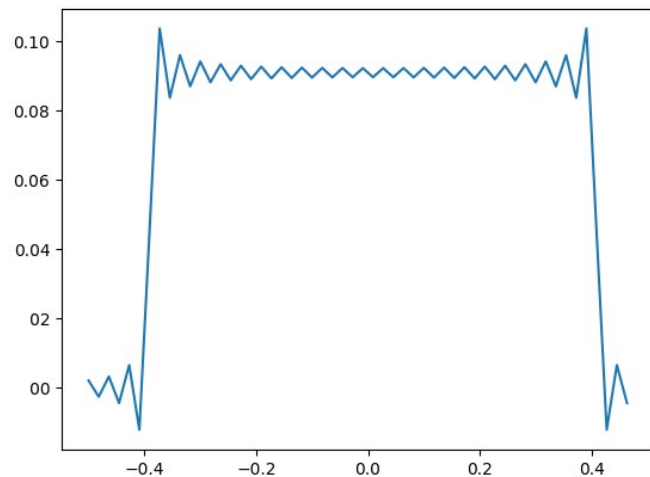
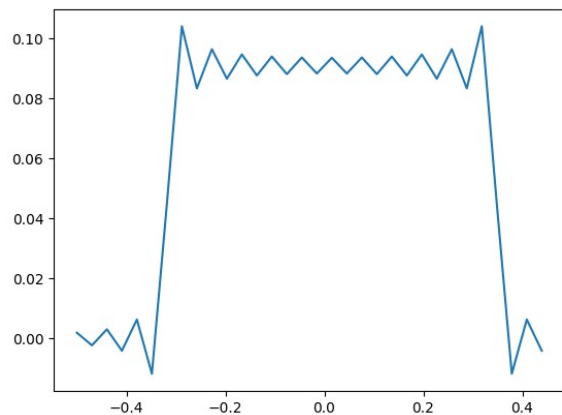
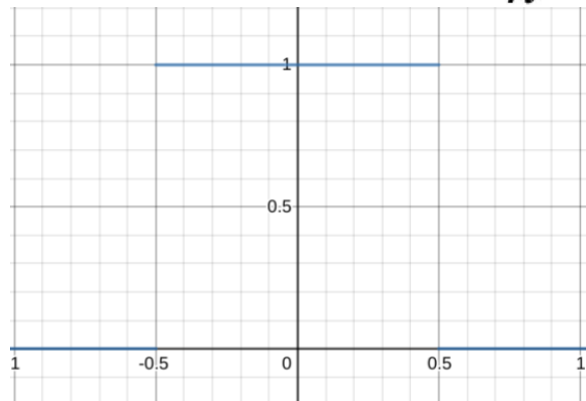
$h' = h/2$



Восстанавливаем функцию

Восстанавливаем сигнал по $\xi'_h(j)$ с помощью теоремы Котельникова–Шеннона:

$$\xi(t) = \sum \xi_h(j) \frac{\sin\left(\frac{\pi}{h}(t - hj)\right)}{\frac{\pi}{h}(t - hj)} \cdot \quad h' = h/2$$



$h' = h/4$